Blind Channel Estimation and MMSE Equalization Using Shalvi and Weinstein's Blind Deconvolution Criteria

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***Abstract*—This paper proposes a novel noise-insensitive approach to blind channel estimation using Shalvi and Weinstein's blind deconvolution criteria. It is based on the relation between the associated equalizer and the nonblind minimum mean square error (MMSE) equalizer reported by Feng, Hsi and Chi. As a by-product, the design of the MMSE equalizer is also presented without training phase. The proposed approach and the designed MMSE equalizer are then justified through computer simulation.**

I. Introduction

Channel equalization is a crucial signal processing procedure for mitigating the multipath fading and noise effects of communication channels. Through the training phase at the expense of system resources such as bandwidth, the channel and the associated statistical parameters such as noise spectrum can be estimated, and the obtained channel and parameters estimates can be applied to design a variety of equalizers such as the well-known minimum mean square error (MMSE) equalizer [1].

Shalvi and Weinstein [2] proposed a class of blind deconvolution algorithms using two cumulants (higher-order statistics [3]) for equalization of nonminimum-phase linear time-invariant (LTI) channels without the training phase when measurements are non-Gaussian. However, for finite signal-to-noise ratio (SNR), their criteria generally are not able to provide consistent estimates for the channel and associated statistical parameters. In this paper, we show that these estimates needed by such as the MMSE equalizer can be accurately extracted using these criteria even when the SNR is low.

II. Model Assumptions

Assume that $x(n)$, $n = 0, 1, \ldots, N-1$, are the given set of measurements generated as follows:

$$x(n) = u(n) * h(n) + w(n) = \sum_{k=-\infty}^{\infty} h(k)u(n-k) + w(n) \tag{1}$$

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where $h(n)$ is an unknown LTI channel, $u(n)$ is a source signal and $w(n)$ is the measurement noise. Let us make the following assumptions for $h(n)$, $u(n)$ and $w(n)$:

- (A1) $h(n)$ is stable with frequency response $H(\omega) = 0$ for $\omega \in \Omega_H \subset [-\pi, \pi)$.
- (A2) $u(n)$ is zero-mean, independent identically distributed (i.i.d.), non-Gaussian with variance σ_u^2.
- (A3) $w(n)$ is zero-mean white Gaussian with variance σ_w^2, and statistically independent of $u(n)$.

The assumption (A1) indicates that when $\Omega_H \neq \emptyset$ (an empty set), the channel $h(n)$ has zeros on the unit circle and its inverse system is unstable, implying that stable equalizer achieving zero-forcing (ZF) equalization [1] does not exist.

III. Review of Shalvi and Weinstein's Criteria

Let $v(n)$ be an equalizer and $e(n)$ be the corresponding equalized signal as

$$e(n) = x(n) * v(n) = u(n) * g(n) + w(n) * v(n) \tag{2}$$

where

$$g(n) = h(n) * v(n) \tag{3}$$

is the (combined) overall system after equalization. Moreover, let $C_{p,q}\{e(n)\}$ denote the $(p+q)$th-order cumulant of (real or complex) $e(n)$ as

$$C_{p,q}\{e(n)\} = \text{cum}\{\underbrace{e(n), \ldots, e(n)}_{p \text{ terms}}, \underbrace{e^*(n), \ldots, e^*(n)}_{q \text{ terms}}\} \tag{4}$$

where the superscript '*' denotes complex conjugation. Note that $C_{1,1}\{e(n)\} = E\{|e(n)|^2\}$.

Shalvi and Weinstein [2] proposed to find the equalizer $v(n)$ by maximizing

$$J_{p,q}(v(n)) = \frac{|C_{p,q}\{e(n)\}|}{[C_{1,1}\{e(n)\}]^{(p+q)/2}} \tag{5}$$

where both p and q are nonnegative integers and $p+q \geq 3$. The criteria $J_{p,q}$ include Wiggins' minimum entropy deconvolution (MED) criterion [4] and Donoho's MED criteria [5] as special ceases. It was proved in [2] that maximizing $J_{p,q}$ leads to the optimum $V(\omega) = 1/H(\omega)$ except for a scale factor and a linear phase term (i.e. the ZF equalization), provided that SNR $= \infty$ and $\Omega_H = \emptyset$.

On the other hand, Feng, Hsi and Chi [6] analyzed the behavior of $v(n)$ associated with $J_{p,q}$ for finite SNR and $\Omega_H \neq \emptyset$. Some of their analytic results are to be used later by the proposed approach, and thus summarized in the following properties:

(P1) The equalizer $v(n)$ is related to the MMSE equalizer, $v_{\text{MSE}}(n)$, via

$$V(\omega) = \alpha \cdot D(\omega) V_{\text{MSE}}(\omega), \quad \forall \omega \in [-\pi, \pi) \tag{6}$$

where α is a real positive constant,

$$V_{\text{MSE}}(\omega) = \frac{\sigma_u^2 \cdot H^*(\omega)}{\sigma_u^2 \cdot |H(\omega)|^2 + \sigma_w^2} \tag{7}$$

and $D(\omega)$ is the Fourier transform of the sequence

$$d(n) = \{p \cdot \xi(n) + q \cdot \xi^*(n)\} \cdot g(n) \tag{8}$$

in which

$$\xi(n) = \left\{ \sum_k [g(k)]^p [g^*(k)]^q \right\} \cdot [g^*(n)]^{p-1} [g(n)]^{q-1}. \tag{9}$$

(P2) Both the equalizer $v(n)$ and the overall system $g(n)$ are stable for finite SNR regardless of $\Omega_H = \emptyset$ or $\Omega_H \neq \emptyset$, and meanwhile

$$V(\omega) = G(\omega) = 0, \quad \text{for } \omega \in \Omega_H. \tag{10}$$

(P3) The phase response

$$\arg[V(\omega)] = -\arg[H(\omega)] - \omega\tau + \kappa, \tag{11}$$

for $\omega \notin \Omega_H$ where τ and κ are constants. In other words, $G(\omega)$ can be a zero-phase system.

IV. Proposed Approach

Let $a(n)$ be the linear prediction error (LPE) filter associated with the measurements $x(n)$. It is well known [7] that $a(n)$ is causal minimum-phase with the leading coefficient $a(0) = 1$. Moreover, when the order of $a(n)$ is sufficiently large, it is a whitening filter, i.e.

$$|A(\omega)|^2 \propto \frac{1}{S_{xx}(\omega)} = \frac{1}{\sigma_u^2 \cdot |H(\omega)|^2 + \sigma_w^2} > 0 \tag{12}$$

where $S_{xx}(\omega)$ is the power spectrum of $x(n)$.

On the other hand, from (8), (9) and (10), it follows that $D(\omega)$ may equal zero for some $\omega \in \Omega_H$. To illustrate this point, let us consider the case that $g(n)$ is real with

$$G(\omega) = \begin{cases} 1, & |\omega| \leq \omega_c \\ 0, & \omega_c < |\omega| \leq \pi \end{cases} \tag{13}$$

where $0 < \omega_c < \pi/2$, implying that $H(\omega) = 0$ for $\omega \in \Omega_H = \{[-\pi, -\omega_c) \cup (\omega_c, \pi)\}$ by (P2). Then the corresponding $D(\omega)$ for $p + q = 3$ is given by

$$D(\omega) = \begin{cases} \frac{\beta}{2\pi} \cdot (-|\omega| + 2\omega_c), & |\omega| \leq 2\omega_c \\ 0, & 2\omega_c < |\omega| \leq \pi \end{cases} \tag{14}$$

(by (8) and (9)) where β is a constant. Clearly, $D(\omega) = 0$ for $\omega \in \{[-\pi, -2\omega_c) \cup (2\omega_c, \pi)\} \subset \Omega_H$.

Let $\Omega_D = \{\omega | D(\omega) = 0\}$. Then, from (6), (7) and (12), we can easily obtain

$$|H(\omega)| \propto \frac{|V(\omega)|}{|A(\omega)|^2} \cdot \frac{1}{|D(\omega)|}, \quad \text{for } \omega \notin \Omega_D \tag{15}$$

Invoking (15) and (P3), we propose the following FFT-based iterative algorithm for estimating $H(\omega)$.

Blind Channel Estimation (BCE) Algorithm

(S1) With finite data $x(n)$, obtain an L_vth-order causal FIR equalizer $\widehat{v}(n)$ using $J_{p,q}$, and an L_ath-order LPE filter $\widehat{a}(n)$ using the computationally efficient Burg's algorithm [7]. Compute their L-point DFT's $\widehat{V}(\omega_k)$ and $\widehat{A}(\omega_k)$ where $\omega_k = 2\pi k/L$, and then compute $R(\omega_k) = |\widehat{V}(\omega_k)|/|\widehat{A}(\omega_k)|^2$.

(S2) Set $i = 0$. Choose an initial guess $|H^{[0]}(\omega_k)|$ for $|H(\omega_k)|$.

(S3) Set $i = i + 1$. Compute $G^{[i-1]}(\omega_k) = |H^{[i-1]}(\omega_k)| \cdot |\widehat{V}(\omega_k)|$ (by (P3)) and then compute its L-point inverse DFT $g^{[i-1]}(n)$. Compute $d(n)$ using (8) and (9) with $g(n) = g^{[i-1]}(n)$ and then compute its L-point DFT $D(\omega_k)$.

(S4) Compute

$$|\widetilde{H}(\omega_k)| = \begin{cases} R(\omega_k)/|D(\omega_k)|, & \text{if } |D(\omega_k)| > \epsilon_D \\ 0, & \text{otherwise} \end{cases} \tag{16}$$

(by (15)) where $\epsilon_D \geq 0$ is a preassigned small constant, and then compute

$$|H^{[i]}(\omega_k)| = |\widetilde{H}(\omega_k)| \Big/ \sqrt{\sum_k |\widetilde{H}(\omega_k)|^2}. \tag{17}$$

(S5) If $\sum_k [|H^{[i]}(\omega_k)| - |H^{[i-1]}(\omega_k)|]^2 > \epsilon_H$ (a preassigned tolerance for convergence), then go to (S3); otherwise, $H(\omega_k)$ is estimated as

$$\widehat{H}(\omega_k) = |H^{[i]}(\omega_k)| \cdot \exp\{-j \arg[\widehat{V}(\omega_k)]\} \tag{18}$$

(by (11)) up to a scale factor and a time delay.

After $\widehat{H}(\omega)$ is obtained by the proposed BCE algorithm, the MMSE equalizer $v_{\rm MSE}(n)$ can be readily estimated via

$$\widehat{V}_{\rm MSE}(\omega) = \widehat{H}^*(\omega) \cdot |\widehat{A}(\omega)|^2 \quad \text{(by (7) and (12))} \qquad (19)$$

up to a scale factor and a time delay. Next, let us show some simulation results.

V. Simulation Results

This section provides two examples to demonstrate the efficacy of the proposed BCE algorithm for channel estimation as well as the design of MMSE equalizer.

Example 1. Broadband Channel

In this example, the source signal $u(n)$ was assumed to be a 4-PAM signal with unity variance and the noise $w(n)$ was real white Gaussian. The channel $h(n)$ was a causal FIR filter with coefficients {0.04, −0.05, 0.07, −0.21, −0.5, 0.72, 0.36, 0, 0.21, 0.03, 0.07} (taken from [1]). In (S1) of the BCE algorithm, $\widehat{v}(n)$ associated with $J_{2,2}$ ($p = q = 2$) and $\widehat{a}(n)$ were obtained with $L_v = 20$ and $L_a = 16$. Then, $\widehat{H}(\omega)$ was obtained by the remaining steps of the BCE algorithm with the initial guess $|H^{[0]}(\omega_k)| = 1$, FFT length $L = 1024$, and the parameters $\epsilon_D = 0.1$ and $\epsilon_H = 10^{-5}$. Thirty independent runs were performed to obtain thirty $\widehat{H}(\omega)$ and $\widehat{V}_{\rm MSE}(\omega)$ (using (19)), and their respective averages, denoted $\overline{H}(\omega)$ and $\overline{V}_{\rm MSE}(\omega)$, were computed. For comparison, the average, $\overline{V}(\omega)$, of the obtained thirty $\widehat{V}(\omega)$ was also computed.

The simulation results are shown in Fig. 1 and 2 where scale factors and time delays were artificially removed. From Fig. 1(a) and 1(b), we can see that the magnitude response and impulse response of both $\overline{H}(\omega)$ and $1/\overline{V}(\omega)$ are quite close to those of $H(\omega)$ for SNR = 20 dB where the dashed line and dotted line in Fig. 1(b) almost overlap each other. On the other hand, as exhibited by Fig. 2(a) and 2(b), the magnitude response and impulse response of $\overline{H}(\omega)$ are close to those of $H(\omega)$ and better than those of $1/\overline{V}(\omega)$ for SNR = 5 dB. These results indicate that $\widehat{H}(\omega)$ obtained by the BCE algorithm is robust against Gaussian noise even for the low SNR case (SNR = 5 dB). Note that the BCE algorithm spent 2 iterations in obtaining $\widehat{H}(\omega)$ for each run of this example.

In addition, as exhibited by Fig. 1(c) and 2(c), not only $\widehat{v}_{\rm MSE}(n)$ but also $\widehat{v}(n)$ can be viewed as a good approximation to $v_{\rm MSE}(n)$, so the latter can also be used as an accurate estimate of $v_{\rm MSE}(n)$ regardless of SNR.

Example 2. Narrowband Channel

In this example, the source signal $u(n)$ was assumed to be a Bernoulli-Gaussian (B-G) sequence [8] as

$$u(n) = u_{\rm B}(n) \cdot u_{\rm G}(n)$$

where $u_{\rm B}(n)$ is a Bernoulli sequence with parameter $\lambda = 0.05$ and $u_{\rm G}(n)$ is a real white Gaussian noise with variance $\sigma_{\rm G}^2 = 0.0225$. The channel $h(n)$ was considered as a real minimum-phase ARMA(4,2) narrowband system taken from [8], and the noise $w(n)$ was real white Gaussian. In (S1) of the BCE algorithm, $\widehat{v}(n)$ associated with $J_{2,2}$ and $\widehat{a}(n)$ were obtained with $L_v = 40$ and $L_a = 30$. The remaining setups for this example were the same as those in Example 1.

The simulation results are shown in Fig. 3 and 4. From Fig. 3(a), 3(b), 4(a) and 4(b), we can see that $\widehat{H}(\omega)$ is an accurate estimate for $H(\omega)$ for both SNR = 30 dB and 10 dB, whereas $1/\widehat{V}(\omega)$ fails to provide reliable channel estimate even for the high SNR case (SNR = 30 dB). Moreover, as exhibited by Fig. 3(c) and 4(c), both $\overline{v}_{\rm MSE}(n)$ and $\overline{v}(n)$ approximate $v_{\rm MSE}(n)$ well. These results again support that the proposed BCE algorithm is effective for both high and low SNR, and indicate that both $\widehat{v}_{\rm MSE}(n)$ and $\widehat{v}(n)$ are good approximations to $v_{\rm MSE}(n)$. As a final remark, the BCE algorithm spent 2 iterations in obtaining $\widehat{H}(\omega)$ for each run of this example.

VI. Conclusions

We have presented a computationally efficient BCE algorithm for the estimation of any arbitrary LTI channel $H(\omega)$ followed by some simulation results to support its efficacy. As a by-product of the proposed BCE algorithm, the MMSE equalizer estimate $\widehat{V}_{\rm MSE}(\omega)$ can also be obtained immediately via (19) without the training phase. Moreover, these results lead to the conclusion that Shalvi and Weinstein's blind deconvolution criteria $J_{p,q}$ can be used to extract channel information (including both magnitude and phase) even for quite low SNR which together with second-order statistical information (power spectra) are needed by other nonblind equalizers.

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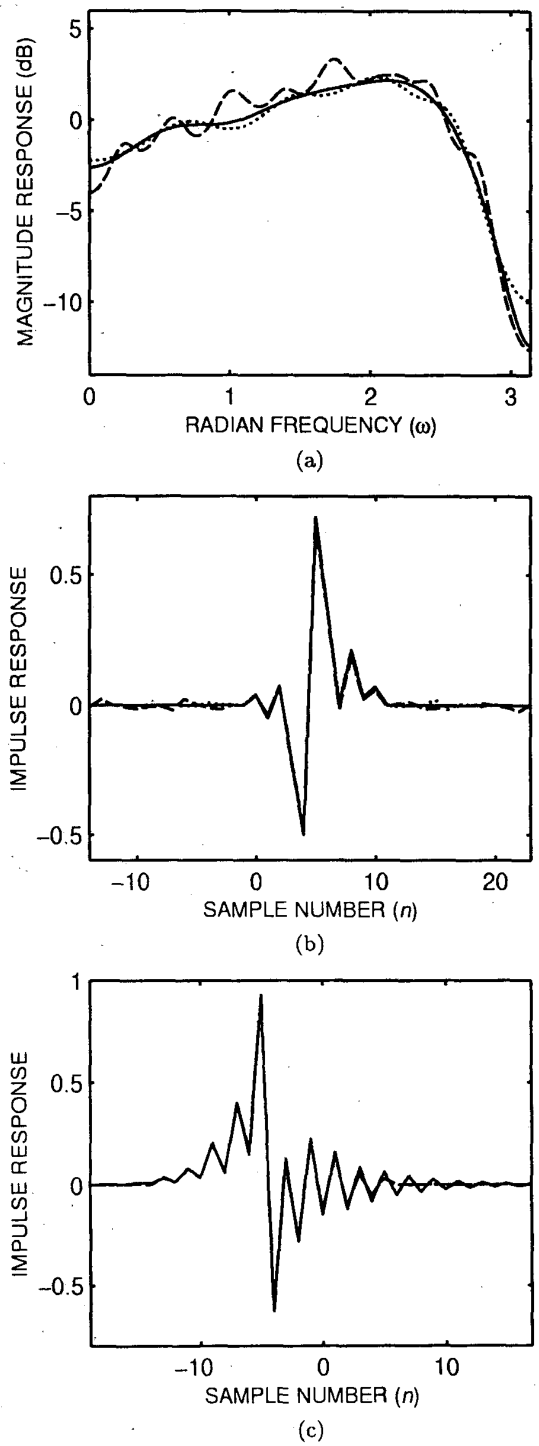

Fig. 1. Simulation results of Example 1 for SNR = 20 dB and $N = 500$. (a) The magnitude response and (b) the impulse response of $H(\omega)$ (solid line), $\overline{H}(\omega)$ (dashed line) and $1/\overline{V}(\omega)$ (dotted line); (c) the impulse response of $V_{\mathrm{MSE}}(\omega)$ (solid line), $\overline{V}_{\mathrm{MSE}}(\omega)$ (dashed line) and $\overline{V}(\omega)$ (dotted line).

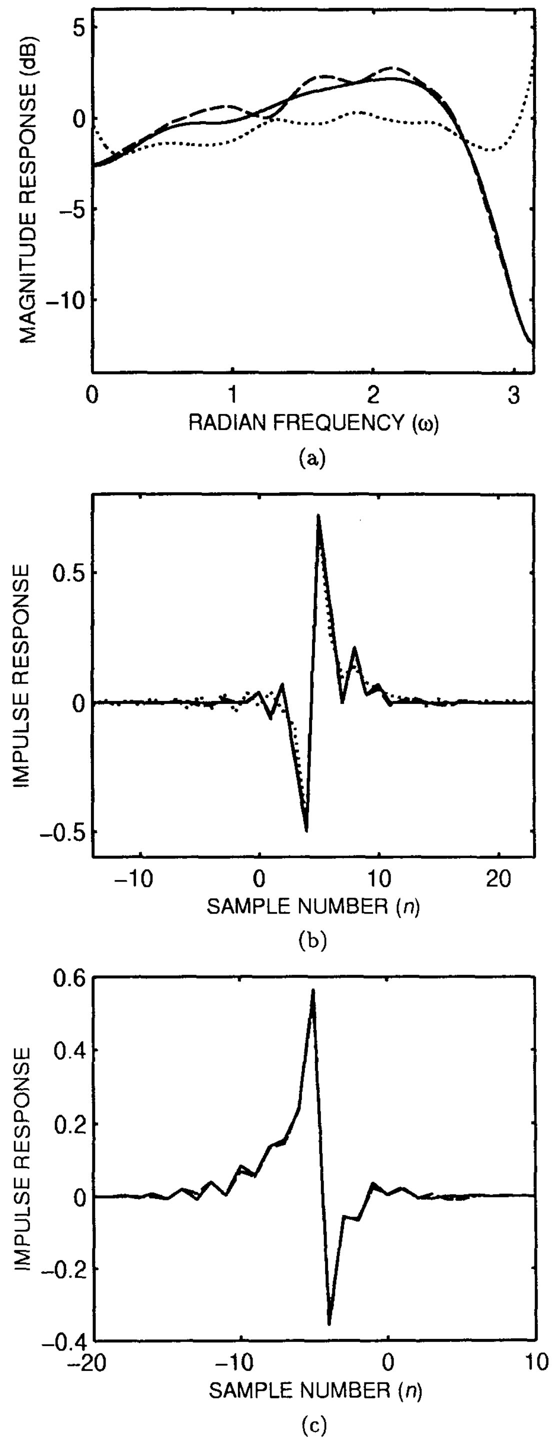

Fig. 2. Simulation results of Example 1 for SNR = 5 dB and $N = 2000$. (a) The magnitude response and (b) the impulse response of $H(\omega)$ (solid line), $\overline{H}(\omega)$ (dashed line) and $1/\overline{V}(\omega)$ (dotted line); (c) the impulse response of $V_{\mathrm{MSE}}(\omega)$ (solid line), $\overline{V}_{\mathrm{MSE}}(\omega)$ (dashed line) and $\overline{V}(\omega)$ (dotted line).

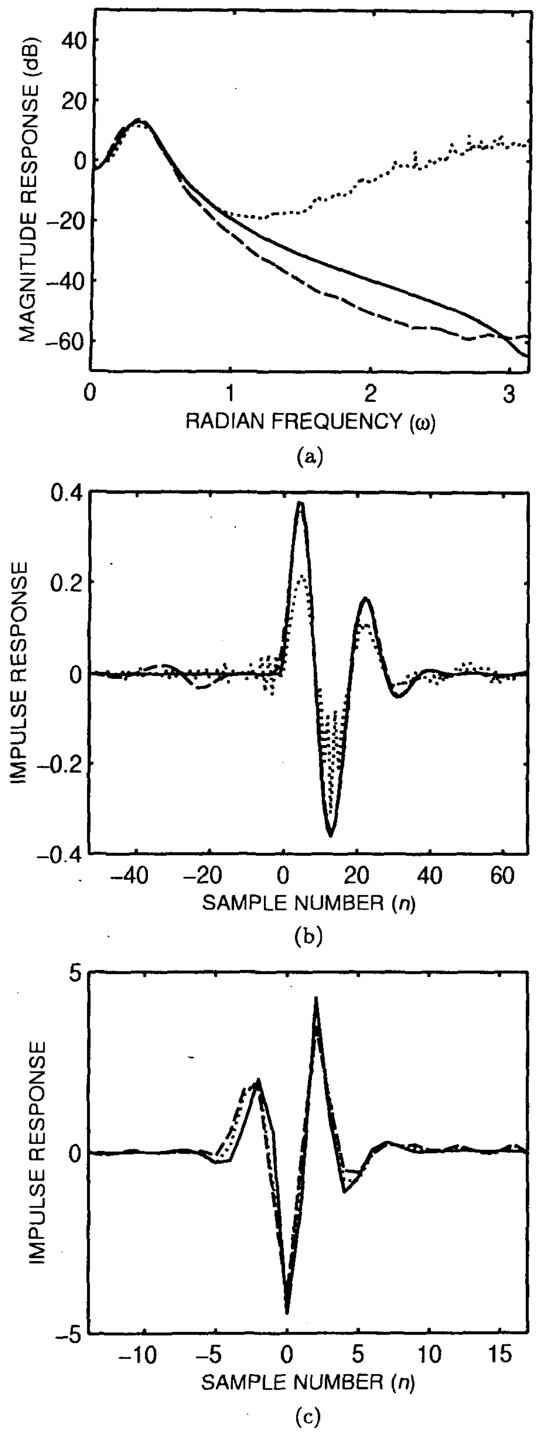

Fig. 3. Simulation results of Example 2 for SNR = 30 dB and $N = 8000$. (a) The magnitude response and (b) the impulse response of $H(\omega)$ (solid line), $\overline{H}(\omega)$ (dashed line) and $1/\overline{V}(\omega)$ (dotted line); (c) the impulse response of $V_{\mathrm{MSE}}(\omega)$ (solid line), $\overline{V}_{\mathrm{MSE}}(\omega)$ (dashed line) and $\overline{V}(\omega)$ (dotted line).

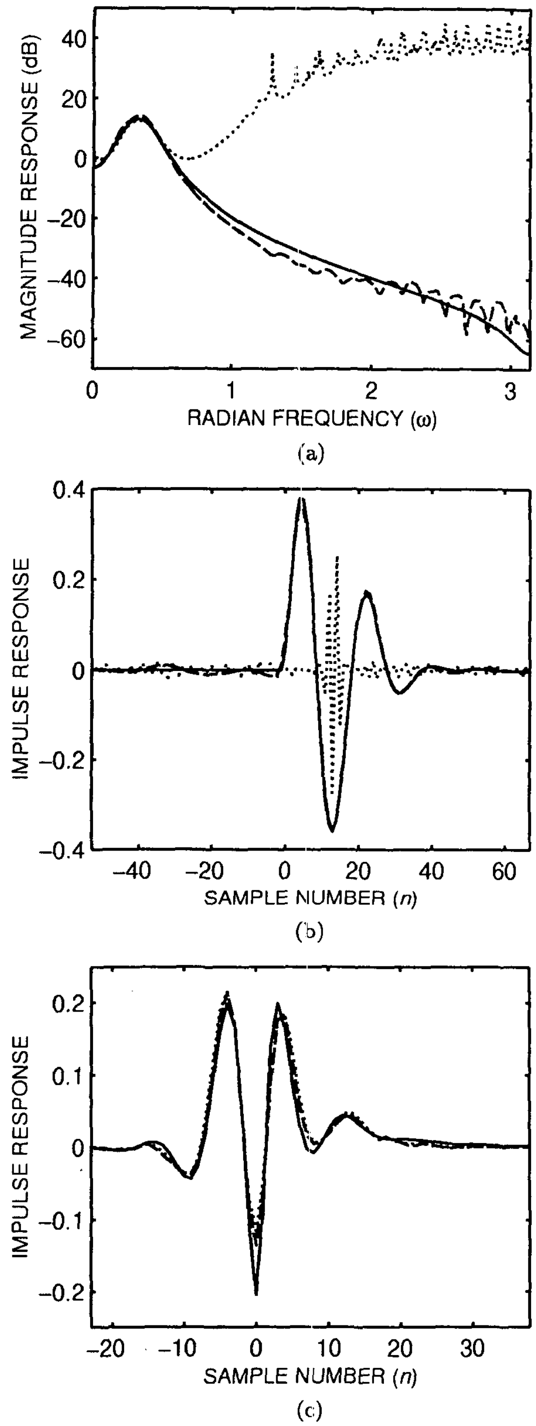

Fig. 4. Simulation results of Example 2 for SNR = 10 dB and $N = 8000$. (a) The magnitude response and (b) the impulse response of $H(\omega)$ (solid line), $\overline{H}(\omega)$ (dashed line) and $1/\overline{V}(\omega)$ (dotted line); (c) the impulse response of $V_{\mathrm{MSE}}(\omega)$ (solid line), $\overline{V}_{\mathrm{MSE}}(\omega)$ (dashed line) and $\overline{V}(\omega)$ (dotted line).